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Crack size independence of the crack driving force in the buckled plate specimen

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A new fracture mechanics specimen, the buckled plate specimen, has been developed with the attractive property that the crack driving force is independent of crack size during constant deflection tests. The mechanics analysis and results of Igepal assisted slow crack growth in polyethylene are presented as verification.

1. Introduction

Great interest exists in the acquisition of slow crack growth (SCG) data for all materials – metals, ceramics and polymers – since it occurs at low stresses, is frequently accelerated by certain environments, and can be the primary cause of failure in engineering structures. Thus there is a practical need to characterize SCG and to understand its mechanisms, so that this understanding can be incorporated into rational engineering design and into material design for improved engineering properties.

Characterization of SCG behaviour in materials by linear elastic fracture mechanics (LEFM) is well established [1–3]. Any pre-cracked specimen, for which the stress intensity factor (K) is known as a function of load (P), crack size (a) and geometry can be used to study SCG, and fracture toughness or both. SCG studies have been done with almost all the common fracture specimens such as single edge notch, compact tension, three-point bend, etc. However, among all the fracture test specimens the most convenient for SCG studies is one in which the crack driving force or the stress intensity factor is independent of crack size. In such a specimen the crack size would be a linear function of time, and the crack velocity would be the slope of the best fit line representing the data. A more elaborate and less accurate data analysis procedure is required for specimens in which K varies as a function of crack size; the data usually have to be curve fit, then differentiated to get the velocity at specific values of K to obtain crack velocity as a function of K.

Two frequently used specimens for which K is independent of crack size are the tapered double cantilevered beam (TDCB) and double torsion (DT) specimens [2–5]. These specimens have been extensively used for SCG studies, in rigid polymers, but they are not suitable for compliant specimens.

A new specimen, the buckled plate (BP) specimen, with crack size independence of G, the crack driving force has been developed which will be particularly useful for compliant materials, thin materials, or in general for specimens of reasonable size and that can be buckled with reasonable loads. This specimen has the following advantages: (1) during constant deflection tests the crack driving force is independent of crack size, (2) it is a simple, flat, ungrooved, rectangular plate, (3) useful with more compliant materials than the DT or TDCB specimens, (4) can be loaded by a simple fixture, therefore does not require a testing machine and can be easily used in hostile environments, and (5) the specimen is ideal for studying the effects of a particular variable, such as environment, temperature, crack driving force, without changing the specimen.

The specimen, the mechanics of crack growth and its use in studying detergent assisted crack growth in polyethylene are described below.

2. Mechanics of the BP specimen

The BP specimen is shown in Fig. 1. If the load on the BP specimen exceeds the critical load for elastic buckling, $P_c = \pi^2 l^{-2} EI$ where $I = wh^3/12$, E is the modulus, and w, h and l are dimensions given in Fig. 1, then the BP specimen can be considered as two equally independent buckled bars as shown in Fig. 2. The usual load–deflection curve after elastic buckling is schematically shown in Fig. 3 by curve AB [6]. Beyond the elastic limit the load–deflection curve would follow BC in Fig. 3, and the analysis that follows would not apply.

Fracture mechanics is founded on the premise that a crack will grow only if the energy released by crack growth is equal or greater than the resistance of the material to crack growth which includes the energy to create new surface, any energy dissipated or consumed by kinetic effects. Therefore, to determine the energy released by crack growth it will be necessary to determine the strain energy in the buckled specimen as a function of load, displacement and geometry.

Following Timoshenko [6] or Williams [7] it can be shown that

$$\frac{\pi^2 P}{P_c} = 4K^2(\beta)$$

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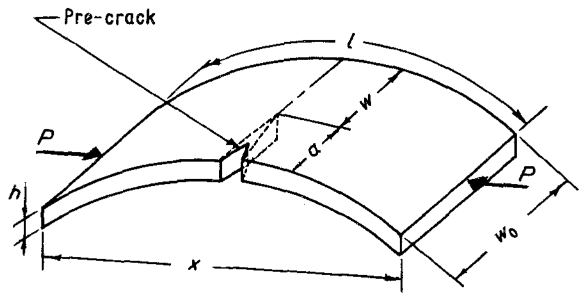

Figure 1 Schematic of loaded BP specimen with dimensions.

and

$$\varepsilon = \frac{l - x}{l} = 2\left(1 - \frac{E(\beta)}{K(\beta)}\right)$$

where $K(\beta)$ and $E(\beta)$ are complete elliptic integrals of the first and second kind, respectively and $\beta = \sin(\alpha/2)$ and x is the chord length of the buckled specimen (see Fig. 2). Therefore, the relation of P to ε can be determined (Fig. 4) and expressed as a polynomial

$$\frac{\pi^2 P}{P_c} = 4.695\varepsilon^2 + 4.523\varepsilon + 9.881 = f(\varepsilon) \quad (1)$$

With this expression the strain energy, SE in an elastically buckled specimen is

$$\text{SE} = l\int_0^\varepsilon P\,\mathrm{d}\varepsilon = \frac{Ewh^3}{12l}\int_0^\varepsilon f(\varepsilon)\,\mathrm{d}\varepsilon$$

or the strain energy density

$$\text{SED} = \frac{\text{SE}}{hlw} = \frac{Eh^2}{12l^2}\int_0^\varepsilon f(\varepsilon)\,\mathrm{d}\varepsilon$$

Following the analysis for a parallel strip or pure shear specimen [8] the strain energy released due to crack growth in a precracked BP specimen can be obtained as follows:

Divide the specimen into four volumes as shown in Fig. 5 then the SED in each volume is

(a) near zero, because the region is relaxed,

(b) unknown, because of uncertainty about the stress and strain field in the crack tip region,

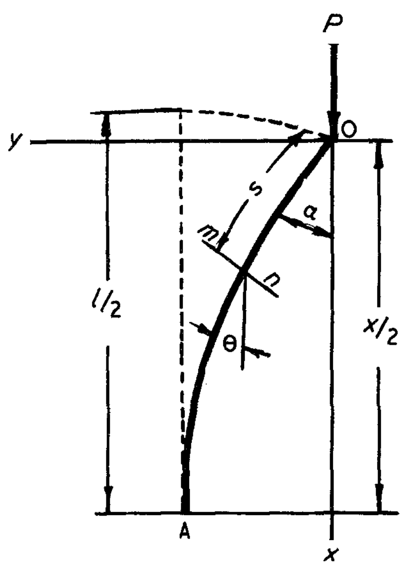

Figure 2 Schematic of buckled bar.

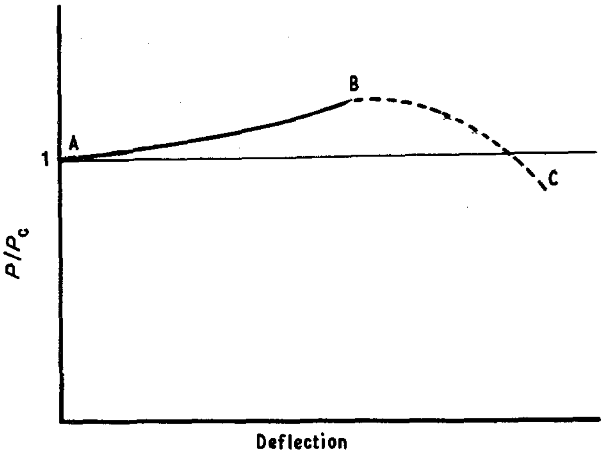

Figure 3 Typical load–deflection curve after elastic buckling.

(c) $Eh^2/12l^2 \int_0^\varepsilon f(\varepsilon)\,\mathrm{d}\varepsilon$

(d) unknown, because of uncertainty about the back edge effects.

Now consider that the crack propagates a small amount da while the displacement is held constant then the following changes occur in the respective volumes:

(a) increased by $hl\,\mathrm{d}a$,

(b) unchanged,

(c) decreases by $hl\,\mathrm{d}a$ and

(d) unchanged.

Therefore, the change in strain energy, due to crack growth da is

$$\text{SED}\; hl\,\mathrm{d}a = \frac{Eh^3}{12l}\left(\int_0^\varepsilon f(\varepsilon)\,\mathrm{d}\varepsilon\right)\mathrm{d}a$$

and the strain energy release rate per unit thickness for an increase in crack length da, G is

$$G = 0.82Eh^2l^{-2}(l - x)f^*(\varepsilon)$$

where

$$f^*(\varepsilon) = 0.158\varepsilon^2 + 0.229\varepsilon + 1 \quad (2)$$

If ε is small then $f^*(\varepsilon)$ is approximately one as shown in Table I. Therefore, the crack driving force G is approximately

$$G = 0.82Eh^2l^{-2}(l - x) \quad (3)$$

which shows that the crack driving force for constant deflection is independent of the crack size.

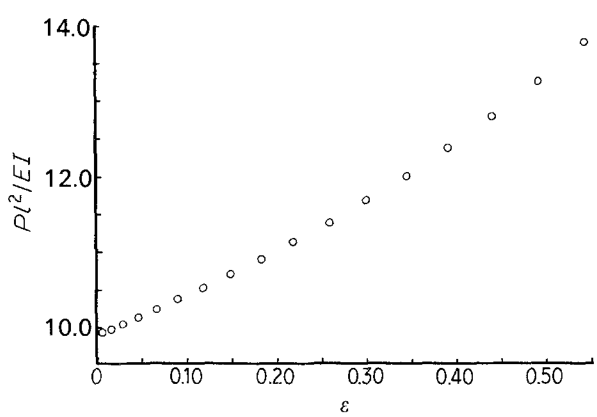

Figure 4 Relationship between load and ε after buckling.

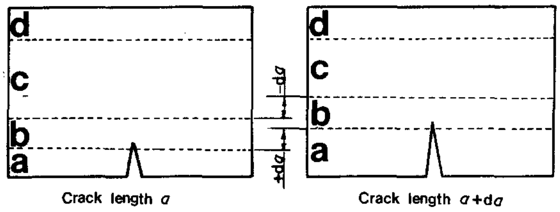

Figure 5 The regions of the BP specimen used to derive the strain energy release rate.

The same result can also be obtained, but in a less transparent manner, through the relation of G to the compliance

$$G = \frac{\mathrm{SE}}{h}\frac{1}{C}\frac{\mathrm{d}C}{\mathrm{d}a}$$

where C is the compliance $(l - x)/P$. Therefore, for a constant displacement test

$$G = -\frac{(l - x)}{h}\frac{\mathrm{d}P}{\mathrm{d}a} \tag{4}$$

since $\mathrm{d}a = -\mathrm{d}w$ and from Equation 1 it also can be shown that

$$G = \frac{\pi^2 E}{12}\frac{h^2}{l^2}(l - x)$$

This derivation, however, provides two valuable results. First, if G is truly independent of crack length then the change of load with crack growth during a constant deflection test should be constant since

$$\frac{\mathrm{d}P}{\mathrm{d}a} = \frac{-hG}{(l - x)} \tag{5}$$

This provides an experimental test of the mechanics analysis. Second, it provides a means for determining the equilibrium modulus of the material which is necessary to calculate G. From the relation for G (Equations 3 and 4) then

$$E = -1.22 l^2 h^{-3}\frac{\mathrm{d}P}{\mathrm{d}a} \tag{6}$$

This is particularly important for tests with polymeric materials because E can be time dependent.

3. Igepal assisted slow crack growth in LDPE

To evaluate the BP specimen the slow crack growth characteristics of low density polyethylene were determined as a function of G in a 10 vol % Igepal CO-630 water solution at 40° C. Igepal CO-630 is the standard ASTM solution for measuring environmental crack resistance of PE. In fact the development of the BP specimen was motivated by results of Ohde and Okamoto [9] on detergent cracking of PE using a bent specimen.

The LDPE was obtained from the Dupont Co, it was compression moulded and was 0.33 cm thick.

TABLE I Relation of $f^*(\varepsilon)$ and ε

ε	0.05	0.10	0.15	0.20	0.25
$f^*(\varepsilon)$	1.011	1.023	1.034	1.046	1.057

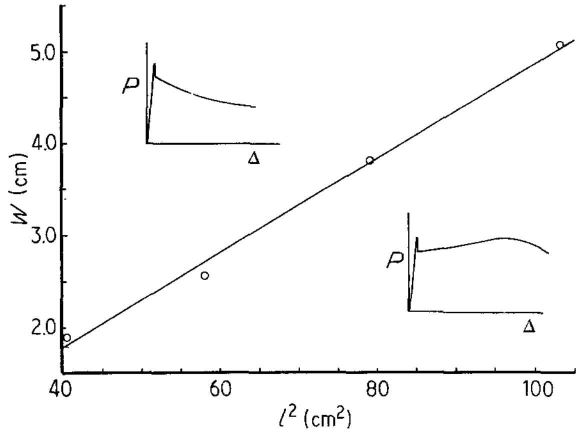

Figure 6 Optimum dimension map for the BP specimen.

4. Optimum specimen dimensions

Valid specimen dimensions are always a concern with fracture mechanics specimens, therefore load–deflection curves were obtained for various combinations of w and l; h was constant. Further work will be necessary to determine the effect of h on valid specimen dimensions. Fig. 6 is a map showing that the characteristics of the load–deflection curve depend on the specimen dimensions. In both regions the load–deflection curve was not like the theoretical elastic buckling curve because of the sharp load drop at the apparent buckling load. Also, there were two types of load–deflection curves: (1) when the dimensions are above the boundary shown in Fig. 6 the load drops with continued deflection, but (2) when the dimensions are below the boundary the load–deflection curve rises with continued deflection. The first region is not in accord with the theoretical predictions, Fig. 3. Therefore, specimens with dimensions in this region were avoided. Also, it was observed that if $w \leqslant 0.03 l^2$ then the load deflection curve increased more rapidly than predicted. Therefore, based on these experimental results for 0.33 thick LDPE the recommended dimensions are $0.03 l^2 \leqslant w \leqslant 0.05 l^2 - 0.29$; and it is also recommended that the critical load be taken as the local minimum.

The dimensions of the specimen used to study Igepal assisted slow crack growth were $8.89 \times 3.18 \times 0.33\,\mathrm{cm}^3$.

5. Specimen preparation and loading

The following experimental procedure gave consistent results:

(1) Round the sharp edges of the specimen.

(2) Mount the specimen in the fixture at the desired deflection.

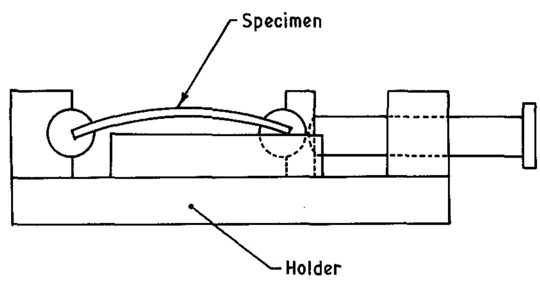

Figure 7 Schematic of the fixture and the specimen.

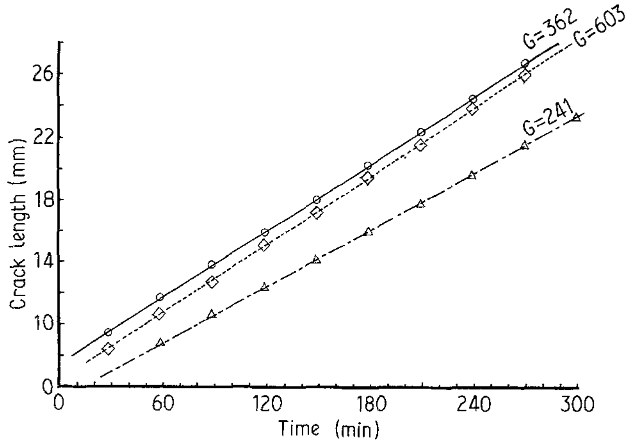

Figure 8 Crack length as a function of time at three different crack driving forces. (△) $G = 241\,\mathrm{N\,m^{-1}}$, (○) $G = 362\,\mathrm{N\,m^{-1}}$, (□) $G = 603\,\mathrm{N\,m^{-1}}$.

(3) Cut a central precrack on one side of specimen at angle of about 40° with a razor.

(4) Immerse the specimen and fixture in the surface active agent.

(5) Measure the crack length with a microscope as a function of time.

Step 1 prevents random crack initiation. The sequence of Steps 2 to 4 shorten the crack incubation period.

A schematic of the fixture is shown in Fig. 7. It was made from Teflon to avoid problems of the fixture interacting with the environment. The fasteners were nylon, because steel fasteners rusted and made observation of the crack difficult. Choice of material for the fixture and its dimensions depend on the system being studied. For the PE/Igepal system the Teflon fixture shown in Fig. 7 worked very well.

6. Crack growth

The theoretical prediction for the BP specimen (Equation 3) that the crack driving force, at constant deflection, is independent of crack size means that the crack velocity should be independent of crack size. Therefore, the prediction can easily be tested by measuring the crack length as a function of time and the relationship should be linear. Fig. 8 shows the crack size as a function of time at three different deflections. All data are straight lines, therefore the velocity is independent of the crack size supporting the mechanics result.

The second prediction from the mechanics analysis was that dP/da should decrease linearly with crack size (Equation 5.). To test this result the fixture was mounted vertically on an Instron testing machine, the specimen buckled, and the load crack size simultaneously recorded as the crack grew. Fig. 9 shows that this is also true; the equilibrium modulus was calculated from the slope of this curve according to Equation 6 and was about 120 MPa. Certainly a reasonable value for LDPE at 40° C.

The effect of the crack driving force, G on crack velocity is shown in Fig. 10. These data are very similar to the results reported by Brown *et al.* [10, 11]. There are three regions: region I where the crack velocity increases with G, region II where the crack velocity is independent of G and region III where the crack decelerates. Brown *et al.* have reported that the deceleration is due to crack branching or blunting; in this material the deceleration was due to blunting, however, in another LDPE material used for some initial tests crack branching was observed. A more detailed report on the characteristics of Igepal assisted crack growth in LDPE as a function of temperature as determined with the BP specimen is in preparation.

7. Crack front profile

As in the DT specimen the crack front during propagation in the BP specimen was not straight, but curved as shown in Fig. 11. The crack front shape was determined by allowing the crack to propagate partly across the specimen then immersing the specimen in liquid nitrogen and rapidly fracturing the cold specimen. The curvature of the crack has been the subject of several analyses which emphasize the distinction between crack speed and velocity [12, 13], these same concerns are relevant to crack growth in the BP specimen. Qualitative observation of the crack shape as a function of driving force indicates that the shape may be a function of the driving force. An analysis and investigation similar to those for the DT specimen is being done.

8. Summary

Provided that the material and dimensions of interest

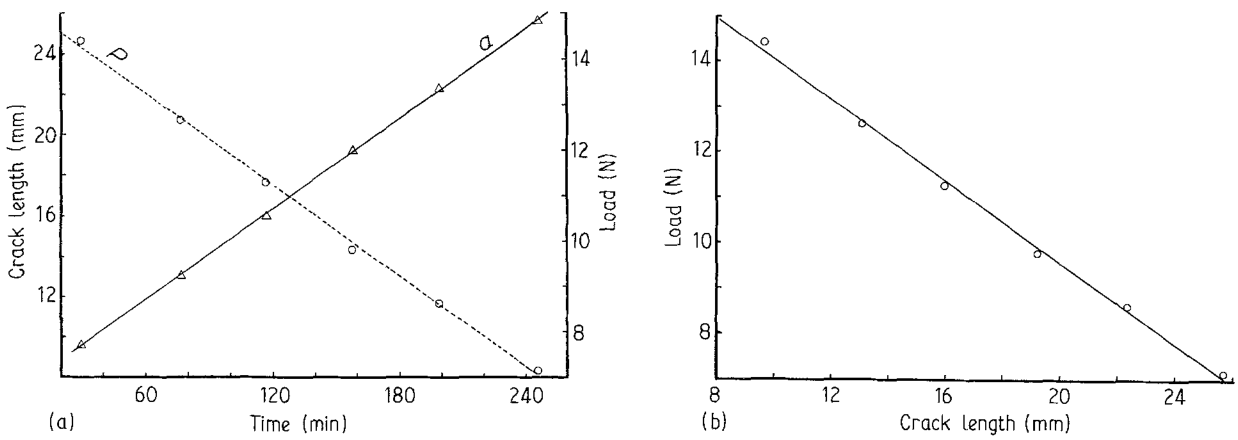

Figure 9 (a) Crack length and load as a function of time, (△) crack length, (○) load. (b) Load relaxation plotted against crack length during crack propagation.

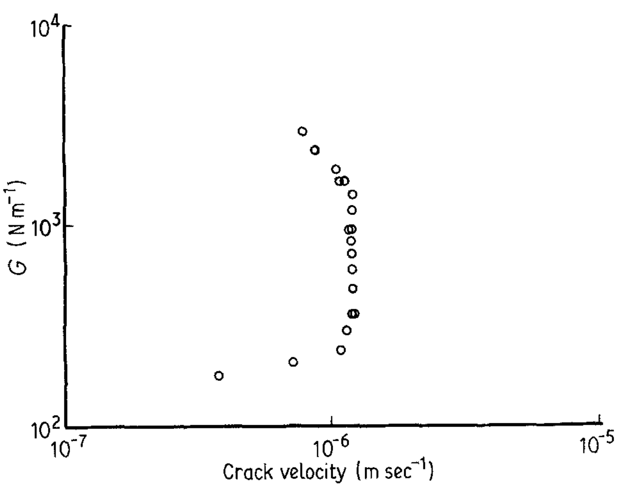

Figure 10 G–a diagram of LDPE at 40° C.

are such that the specimen can be buckled at a reasonable load the BP specimen is a simple, cheap specimen with the very attractive feature that the crack driving force is independent of crack size. The mechanics analysis and experimental results of Igepal assisted slow crack growth in LDPE support this conclusion.

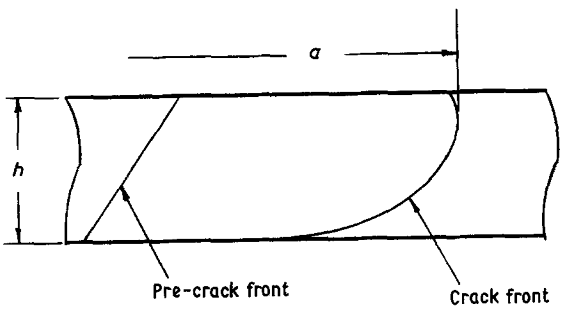

Figure 11 Crack front profile of the BP specimen.

The specimen can be loaded in a fixture, therefore not requiring a testing machine and can be exposed to hostile environments. In theory the specimen should also be useful for determination of the fracture toughness and fatigue crack growth characteristics. Additional research is needed to determine its general validity and applicability, and to understand the consequences of the crack front curvature.

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